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Pulse-To-Pulse Time-Frequency Analysis of an Agile Pulsed Radar Signal Using the Spectrogram Distribution

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A. S. Saliu<sup>1</sup>, A. A. Ahmad<sup>2\*</sup>, A. M. S Tekanyi<sup>3</sup>, O. E. Eguaroje<sup>1</sup>, J.B. Ayeni<sup>2</sup>

<sup>1</sup>National Space Research and Development Agency (NASRDA),  
Federal Ministry of Science and Technology, Abuja, Nigeria

<sup>2</sup>Dept of Electrical and Electronic Engineering,  
Faculty of Engineering, Nigerian Defence Academy (NDA), Kaduna, Nigeria

<sup>3</sup>Department of Communications Engineering,  
Ahmadu Bello University, Zaria, Nigeria

E-mail: [aaashraf@nda.edu.ng](mailto:aaashraf@nda.edu.ng)

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### ABSTRACT

*In recent times, electronic technology plays an increasingly important role in military operations. New forms of radar signal waveforms are constantly being designed for military applications with unique characteristics in order to prevent their detection. As such, better signal processing tool is required in field of electronic warfare support (ES) to analyze the intercepted emitted signal for the detections of its radar capabilities. An algorithm set is developed in this paper based on spectrogram time-frequency distribution (TFD) in order to estimate and classify the time and frequency characteristics of an agile pulsed modulated radar signal. Other related digital signal processing tools such as Hilbert transform, instantaneous frequency (IF) and instantaneous power (IP) were also used in order to achieve the objective of this work. Performance analysis of the estimations was validated using Monte Carlo simulation performed at various ranges of signal-to-noise ratios (SNRs) in the presence of additive white noise of Gaussian probability density function. Results obtained showed probability of correct parameter estimation (POCPE) of 100% is achieved at minimum SNR of -4dB irrespective of the parameter being estimated. Estimation results obtained at this good SNR shows the algorithm developed can be deployed in the practical EW field.*

**Keywords:** Electronic warfare (EW), electron warfare support (ES), time-frequency distribution (TFD), Monte Carlo simulation, signal-to-noise ratio (SNR)

### INTRODUCTION

The electronic warfare support (ES) of the electronic warfare (EW) deals with tactical interception through the passive surveillance of the electromagnetic (EM) spectrum using radar to detect the enemy's position, strength, and intention, besides warning of targets (Neri, 2006). It involves detection, recognition, tracking, targeting, warning, destruction, and avoidance (Elsworth, 2010). A key aspect of the ES involves the process of analyzing radar signals in order to obtain information contained in these signals (Wiley, 2006). Besides the carrier frequency, airborne radar signals can be characterized by time parameters such as pulse repetition interval (PRI) and the pulse width (PW). The signals involved in ES are usually of unique characteristics

different from the non-militarized radar emitter signals, thus making them harder to detect or jam. This signifies the need for research into these problems of radar signal parameter estimation. In recent related research papers, an algorithm for automatic recognition of different radar waveforms was designed using Wigner Distribution and Choi-Williams time-frequency Distributions (WD and CWD). (Lunden *et al*, 2007). Correct classification rate of 98% was achieved at SNR of 6 dB; however, the TFDs used were of higher complexity when compared to the proposed TFD to be used in this work. A parameter estimation based on multi-phase difference method was utilized for estimating a hybrid LPI-based radar signal (Xiong *et al*, 2010). The presented algorithm had higher accuracy of parameter estimation when the SNR was above 11 dB. The proposed method had the advantage of simplicity due to use of fast Fourier transform, however minimum SNR was quite high. Airborne radar signals type analysis and classification (ARTAC) system was designed to determine electromagnetic order of battle (EOB) in the field of ES system (Ahmad *et al*, 2014). The method achieved a classification accuracy of 90% at 6.2 dB; however, accurate frequency parameter estimation was not achieved. Low probability of intercept (LPI) radar signals of specific groups were characterized using time frequency (TF) rate and scalogram respectively. (Li *et al*, 2014; Stevens *et al*, 2016). Recognition rate of 100% was at low SNR of -2dB for the first case while 81% detection was at low SNR of -3 dB for the latter case. Other recent developments in the use of time-frequency analysis for radar-based applications may be found in (Ahmad *et al*, 2015b)

As such, this paper uses the low complexity spectrogram time frequency distribution (TFD) to estimate the agile pulsed radar signal of LPI capabilities considering its non-constant time and frequency characteristics. The agile pulsed radar is capable of changing its time and frequency parameter on pulse-to-pulse basis. The main TFD is preceded by Hilbert transform analysis and succeeded by analyses involving instantaneous power (IP) and instantaneous frequency (IF) to extract the time and frequency parameters. Accuracy of analysis is determined using a developed algorithm based on Monte Carlo simulation performed at various ranges of SNR in the presence of additive white. MATLAB® (R2015a version) is used to realize most part of this research.

The rest of the paper is as follows; key concepts related to this paper were presented in section 2, methodology used was discussed in section 3. Results and accompanying discussions was carried out in section 4 before final conclusion was presented in the last section 5.

### **Brief Overview of Key Concepts**

The word radar stands for radio detection and ranging, which indicates the basic purpose of radar is to detect and determine the location of a target. The detection and ranging are achieved with the aid of transmitter transmitting electromagnetic pulses on the principle of generating an electromagnetic pulse signal, transmits it through a medium and waits for an echo to return (Wolff, 2009). There are two main time characteristics of radar signal; the pulse width (PW) and pulse repetition interval (PRI). The pulse width (PW) refers to the time taken for the transmitter to send out RF energy or the time the radar system takes to radiate each pulse. The PRI is the time between the beginning of one pulse and the start of the next pulse in a specific time unit. These parameters can be used to determine other characteristics of the radar such as

the maximum unambiguous range and range resolution (Ahmad *et al*, 2015a). The carrier frequency of the radar is the operating frequency of radar that carries transmitting information within PW duration. For example, long range radars use lower frequency in order to reach large antennas and high-power transmitters whereas short range radars use otherwise (Skolnik, 2008). The agile pulsed radar signal is a case of where there are both varying time and frequency characteristics. This type of signal is depicted in Figure 1 based on personal MATLAB® coding.

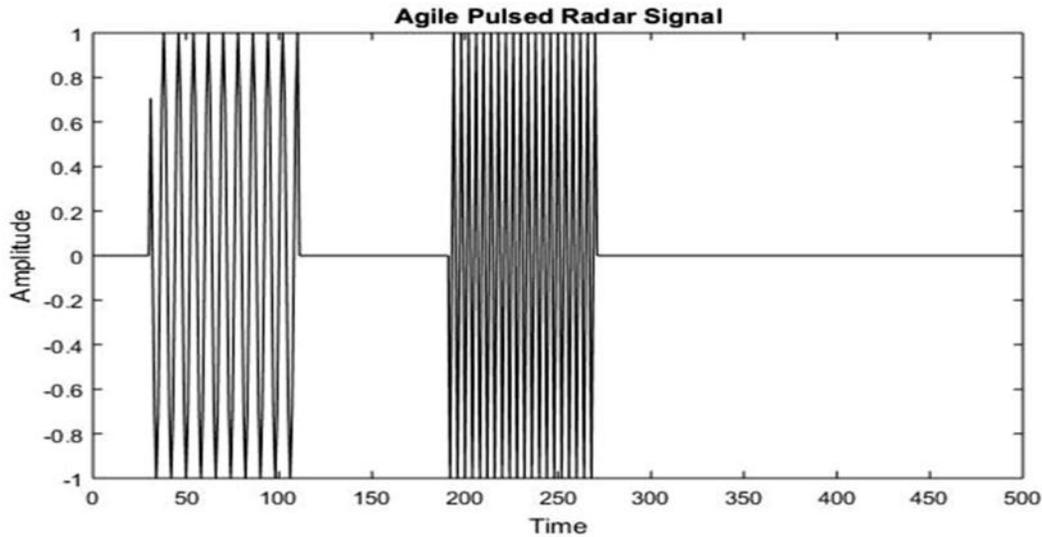


Figure 1: Agile Pulsed Radar Signal

It can be seen in figure 1, that the rate of change in the first pulse is different from that of the second, the second is higher. Similarly, the second PRI is also greater than the first PRI. The spectrogram is a windowed joint time-frequency distribution member of the TFD, which comes in a form of square magnitude of the short time Fourier transform (STFT) (Boashash, 2015). The process of getting spectrogram from the STFT is shown in (1) to (2);

$$SP(t, f) = \left| F_{t \rightarrow f} \left\{ s(\ddagger) w(\ddagger - t) \right\} \right|^2 \tag{1}$$

$$SP(t, f) = \left| \int_{-\infty}^{\infty} s(\ddagger) w(\ddagger - t) e^{-j2f f \ddagger} d\ddagger \right|^2 \tag{2}$$

where:  $F_{t \rightarrow f}$  is taking, a Fourier transform with respect  $\tau$ ,  $s(\ddagger)$  is the signal,  $w(\ddagger)$  is the window and  $\ddagger$  is arbitral time parameter. It should be noted that the observation window allows localization of the spectrum in time, but also ‘smears’ in frequency due to the convolution operation, leading to an always trade-off between time and frequency resolution (Boashash, 2015). Recently, L-statistics approach was used on the spectrogram to remove micro-Doppler (m-D) effect in radar imaging analysis (Stankovic *et al*, 2013). Time-frequency signal analysis requires use of the analytic signal. Signal analysis using analytical signal will have neither any aliasing effect nor frequency artifacts (Boashash, 1988). Hilbert transform is used to introduce a phase lag of 90°. It is related to the actual data by a 90-degree phase shift; sines become cosines (3) and vice versa (4) and forms the integral part of converting a signal to its analytic form.

$$H(\sin 2f ft) = -\cos(2f ft) \tag{3}$$

$$H(\cos 2f ft) = \sin(2f ft) \quad (4)$$

where:  $H$  is, Hilbert transform,  $f$  is frequency and  $t$  is the time

The instantaneous power (IP) can be simply defined as the distribution of the energy of the signal over instants of time through the convolution of signal and its conjugate (Page, 1952). It can either be gotten directly or it can be gotten from a TFD (in this case the spectrogram). This is shown respectively in (5) and (6):

$$IP(t) = s(t) * s^*(t) = |s(t)|^2 \quad (5)$$

$$IP(t) = \max SP(t, f) \quad (6)$$

where:  $s^*(t)$  is the complex conjugate of the signal,  $s(t)$  and  $SP(t, f)$  is the obtained spectrogram representation. The instantaneous frequency (IF) is used to track the frequency of a signal of interest at specific time instances. It is gotten from the peaks of the spectrogram time-frequency representation (TFR) along the frequency axis and can be mathematically expressed in (7) (Boashash, 2015):

$$IF(t) = \max_f SP(t, f) \quad (7)$$

where  $SP(t, f)$  is the obtained spectrogram representation. IF has several advantages which include, easy implementation, easy to use, and no cross-terms. However, it also has disadvantage associated with resolution just like the spectrogram due to the uncertainty principle of time and frequency. Recently, its estimation for multi-component signals using locally adaptive TFD was carried out (Khan *et al*, 2015) and its measurement was done using four-wave mixing method for a 40 GHz bandwidth waveguide (Pagani *et al*, 2016). It can be seen that the main difference between (6) and (7) is frequency ( $f$ ) below the max, indicating the IP is simply the power obtained at the peak of the spectrogram for which the IF is the frequency gotten at instants of time during the peak obtainment.

## METHODOLOGY

The first step taken in the algorithm designed for generation of the signal is converting of time parameters from seconds (precisely microseconds for radar signals) to samples using the sampling frequency based on the basic signal processing formula shown in (8);

$$t = NT_s = \frac{N}{F_s}; \therefore N = t * F_s \quad (8)$$

Where  $t$ =time in seconds;  $N$ = time in samples;  $T_s$  = sampling time and  $F_s$  = sampling frequency. Thereafter, PW part of signals is generated using a co-sinusoidal command at each carrier frequency for the duration of stipulated PW time and PRI part is gotten using 'zeros' command for the duration of the stipulated PRI times. The 'zeros' command in MATLAB is used to generate a matrix of zeros and was also used to generate delay time (of 30 samples) at the beginning of the signal in order to simulate practical scenario of an incoming radar signals.

The main estimation algorithm firstly involved the conversion of incoming signal to analytic or complex format using Hilbert transform in order to eliminate the non-required mirror frequency content. It is assumed that this signal has been de-interleaved and down converted to intermediate frequency. De-interleaving is a field of ELINT on its own that allows this work to consider the signal as being from a specific radar emitter while the down conversion allows us to

work in MHz as opposed to GHz used in antenna, similar to other communication systems. The Hilbert transform is inbuilt already in the MATLAB was called upon in this section in order to achieve the analytic conversion. However, it is important to point out MATLAB version of the Hilbert transform command is a step further than normal phase shift of the transform as it gives an analytic output format directly from the input format. Thereafter, spectrogram TFD was used to analyse the time and frequency content of the signal in order to obtain its time-frequency representation (TFR). Basically, its code was constructed based on two inputs (the signal and window length) and give a single spectrogram output containing power, frequency and time content of the input signal.

The Bartlett window (triangular) was used for spectrogram due to its high peak side lobe as compared to other basic window functions and hence its superiority and a moderate window length of 64 samples were used in order to avoid the smearing problem of spectrogram. The 3D and 2D plots of the main test signal is illustrated in figure 2. The 3D has time, frequency and power domains while the 2D is just time and frequency domains.

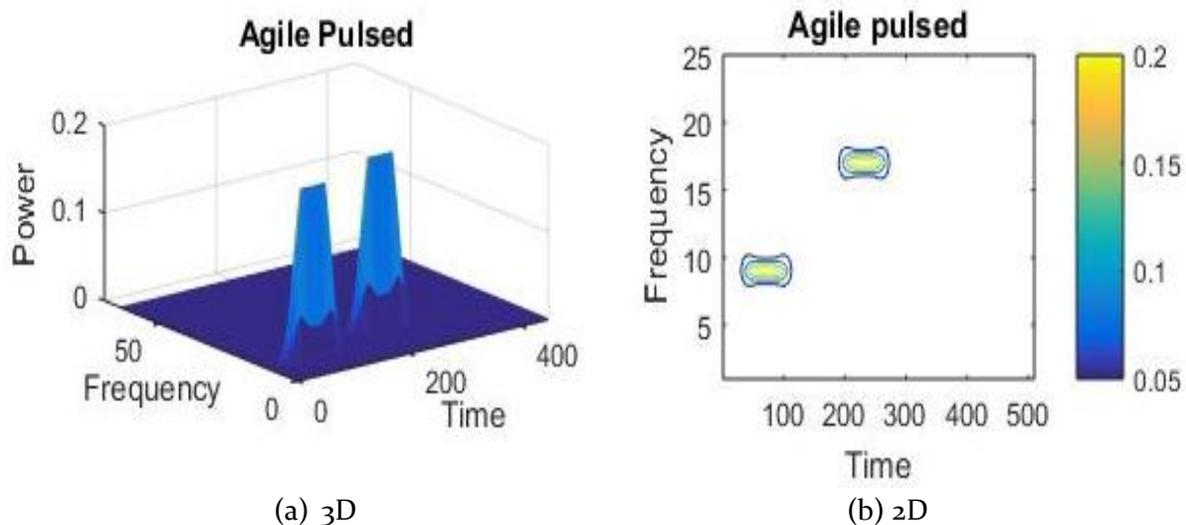


Figure 2: 3D and 2D TFR plots of the radar signals using spectrogram

The colored side bar in 2D plot of figure 2 represents the level of power. Examination of figure 2 show the Agile pulsed has varying carrier frequencies and PRI. Finally, IP and IF based on equations (6) and (7) were estimated from the TFR shown in figure 2 for time and frequency parameters estimation respectively. Both of these parameters were gotten at instants of time, where IP corresponds to maximum power based on the search along the frequency axis while IF corresponds to the actual frequency corresponding to this maximum power along this axis search. These steps are obtained by the combination of FOR loop to control the search and inbuilt MATLAB 'max' function to find the maximum power. These estimations allow for conversion of output signal from the time-frequency analysis of 3D to 2D of power and time for IP and frequency and time for IF.

A simple sub-algorithm was designed to estimate twice when the signal is high for PW estimation and low for PRI estimation. Carrier frequencies were estimated at each PW through finding the average value of frequency at instants of time for the PW duration. Finally, these

obtained parameters from the aforementioned analysis are converted back to microseconds in case of time parameters and Megahertz by the use of formula sets which relates samples using sampling frequency. These formulas are presented in (9) and (10).

$$T = \frac{N_t}{F_s} \quad (9)$$

$$F_c = \frac{F_s}{N_s} N_f \quad (10)$$

Where  $T$  is the time parameter in seconds,  $N_t$  is the time parameter in samples,  $F_s$  is the sampling frequency,  $F_c$  is the carrier frequency,  $N_s$  is the window length for TFD used (Spectrogram) and  $N_f$  is the frequency parameter in samples. The time parameter can be either the PW or PRIs while the carrier frequency can be of the first or second pulse.

## RESULTS AND DISCUSSION.

For analysis of this received signal, it assumed that the signal is de-interleaved and down converted to achieve a current standard sampling frequency of 100 MHz based on Agilent P-Series Power meters for radar signal surveillance (Agilent, 2014). For performance analysis; the noise environment is modeled by the additive white noise. This noise is very common and suitable for microwave communications environment considered in this paper. The estimation performance is done through a Monte Carlo simulation setup whereby two loops controlling the signal-to-noise ratio (SNR) and iteration are used. A standard iteration of 100 loops for each SNR is used in this work. The SNR is simply the ratio of the signal power to the noise power obtained through the use of variance. The test agile pulsed radar signal used had a PW of  $1\mu s$ , first and second PRI of  $40\mu s$  and  $100\mu s$ , first and second carrier frequency of 10MHz and 20 MHz.

The PW of  $1\mu s$  is used due to 'simple' sinusoidal intrapulse (internal) modulations, PRI of  $40\mu s$  and  $100\mu s$  corresponds to small and medium range of 6Km and 15Km respectively based on example in (Stimson, 1998). Carrier frequencies are selected within the Nyquist sampling theory i.e. carrier frequencies must be less than or equal to half the sampling frequency to avoid aliasing (Shenoi, 2006). Probability of correct parameter estimation (POCPE) for each parameter of the signal is obtained after generation of test signals and addition of AWGN for SNR range of -18dB to 17dB. The POCPE is ratio of the correct estimation based on parameter used to generate the signal to the total standard number of 100 estimations. The POCPE is expressed in percentage. The obtained results for time parameters POCPE for the test signals considered in tabular form is given in table 1:

**Table 1:** Monte Carlo Simulation Results for POCPE (Time)

SNR (dB)	POCPE for PW (%)	POCPE for PRI <sub>1</sub> (%)	POCPE for PRI <sub>2</sub> (%)
-18	85	5	0
-17	96	72	29
-16	100	99	88
-15	100	100	93
-14	100	100	99

SNR (dB)	POCPE for PW (%)	POCPE for PRI <sub>1</sub> (%)	POCPE for PRI <sub>2</sub> (%)
-13	100	100	99
-12	100	100	100
-11	100	100	100
-10	100	100	100
-9	100	100	100
-8	100	100	100
-7	100	100	100
-6	100	100	100
-5	100	100	100
-4	100	100	100
-3	100	100	100
-2	100	100	100
-1	100	100	100
0	100	100	100
1	100	100	100
2	100	100	100
3	100	100	100
4	100	100	100
5	100	100	100
6	100	100	100
7	100	100	100
8	100	100	100
9	100	100	100
10	100	100	100
11	100	100	100
12	100	100	100
13	100	100	100
14	100	100	100
15	100	100	100
16	100	100	100
17	100	100	100

It can be observed from table 1 that different time parameters have POCPE of 100% at different SNR. POCPE of 100% is achieved irrespective of the time parameter in consideration from a specific SNR. It can also be deduced from table 1 that POCPE of 100% for PW and PRI<sub>1</sub> are very close compared to PRI<sub>2</sub> for all the signals considered, i.e. a difference of 1dB between PW and PRI<sub>1</sub> estimation is noted for this agile pulse radar signal. POCPE of 100% is noted for PRI<sub>2</sub> after a difference of 3dB at SNR of -12dB. This replicate noise effect of practical ES scenario where the

noise effect is higher as the signal gets longer. As such, at SNR of -12dB, POCPE of 100% is attained for all time parameters irrespective of the signal in consideration, and hence marks the minimum SNR required for completely accurate time parameter estimation using this methodology. This table and above points are further illustrated by presentation of the parameters plot diagram in the figure 3.

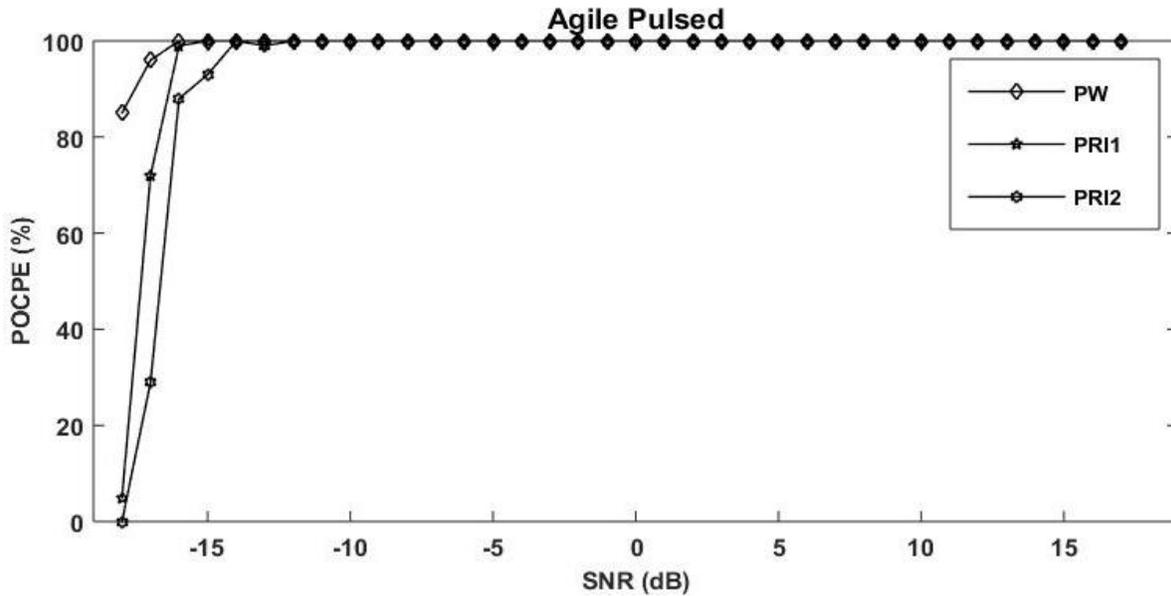


Figure 3: Monte Carlo simulation results for time parameters POCPE of Agile pulsed radar signal

It is clearly seen in figure 3 that different time parameters have a different POCPE of 100% for this radar signal. This can be attributed to the random nature of the noise model of AWGN modeled to mimic the uncertainties in the real world. Similarly, the obtained results for frequency parameters POCPE for the test agile pulsed radar signal in tabular form is given table 2:

Table 2: Monte Carlo Simulation Results for POCPE (Frequency)

SNR (dB)	POCPE for F <sub>1</sub> (%)	POCPE for F <sub>2</sub> (%)
-18	99	23
-17	99	52
-16	100	73
-15	100	80
-14	100	83
-13	100	86
-12	100	84
-11	100	91
-10	100	92
-9	100	94
-8	100	96

SNR (dB)	POCPE for F <sub>1</sub> (%)	POCPE for F <sub>2</sub> (%)
-7	100	98
-6	100	99
-5	100	99
-4	100	100
-3	100	100
-2	100	100
-1	100	100
0	100	100
1	100	100
2	100	100
3	100	100
4	100	100
5	100	100
6	100	100
7	100	100
8	100	100
9	100	100
10	100	100
11	100	100
12	100	100
13	100	100
14	100	100
15	100	100
16	100	100
17	100	100

Similar to results obtained in table 1 for estimations of time parameters, it can be seen from table 2 that different frequency parameters have POCPE of 100% at different SNR and that from a specific minimum SNR; POCPE of 100% is achieved irrespective of the frequency parameter being estimated. Two other observations can also be made when table 2 is examined; firstly, the POCPE of 100% for carrier frequency of first pulse (F<sub>1</sub>) differs from that of second pulse (F<sub>2</sub>) with SNR difference of 12dB (-16dB and -4dB) noted. This also replicates noise effect of practical ES scenario where the noise effect is higher as the signal gets longer. Secondly, at SNR of -4dB, POCPE of 100% is attained for all frequency parameters irrespective the frequency parameter being estimated. Table 2 and above points are further illustrated by presentation of the frequency parameters estimation plot diagram in the figure 4.

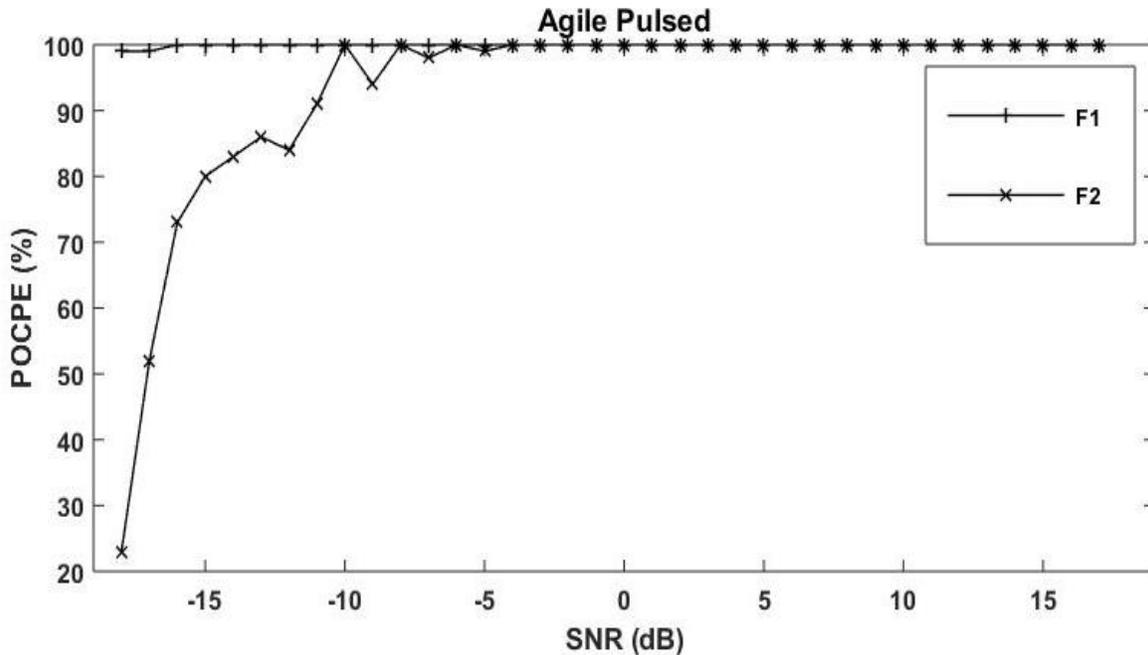


Figure 4: Monte Carlo Simulation Results for frequency parameters POCPE of agile pulsed radar signal

It is clearly seen from figure 4 that different frequency parameters have different POCPE of 100% for different radar signals. This can also be attributed to the random nature of the noise model of using AWGN which also illustrates the randomness of the noise in the real world. It can be generally stated that the minimum SNR required for frequency parameters estimation is higher than that of time parameters estimation due to the higher samples of the latter.

## CONCLUSION

The main tool of analysis used in this paper is the spectrogram TFD to obtain the signal TFR of varying pulse-to-pulse time and frequency parameters. Hilbert transform was first used to cater for Fourier transform limitation while IP and IF are obtained from this TFR in order to estimate time and frequency parameters correctly. Monte Carlo simulation was set up for parameter estimation in the presence of additive white noise within specific SNR for performance analysis of the algorithm. Parameters whether time or frequency, were found to have POCPE of 100% at different minimum SNR. Parameters associated with second pulse (PRI<sub>2</sub> and second carrier frequency) were found to require a higher SNR to achieve POCPE of 100% as compared to those associated with first pulse due to effect of noise being higher along the signal time length. Future works will involve the use of more sophisticated radar signal configuration, where signal with other forms of PRI modulations such as jittering, sliding, dwell and switching, etc would be considered.

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